University of Baghdad
College of Engineering
Civil Engineering Department
Answer 5 Questions Only
Instructors: Dr. Abbas A. & Qusay A.

Time: 3 hours
Date: June, 12, 2013
First Attempt

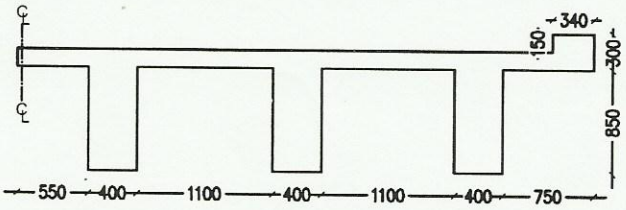
SELECTED TOPICS

Q1) For the simply supported reinforced concrete girder-deck bridge shown in fig. , having the following data:

Clear span: 15 m $f_c' = 25$ MPa
Span C/C: 16 m $f_y = 400$ MPa
Truck: MS 18 Asphalt wt. = 1.5 kN/m^2
handrail wt. = 1.2 kN/m , Find:

a-Slab total moment.

b- Shear reinforcement at the critical section of an exterior girder (use ∅16 bars).(Let d = h – 150)



Bridge cross-section (all Dimensions in mm)

Q2) For the simply supported one-way concrete slab bridge having the following data:

Clear span: 7 m Truck: MS 18 asphalt wt. = 1 kN/m^2
Span C/C: 7.5 m Slab thickness: 400 mm edge beam total depth = 600 mm
Edge beam width = 500 mm handrail wt. = 1 kN/m^2
Find:

a- Slab total moment.

b- Edge beam total moment.

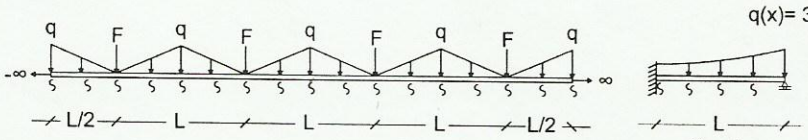
Q3) For the beams on elastic foundation shown in figs.,

a- Choose a suitable coordinate system.

b- State the complementary solution.

c- Find the particular solution .

d- State the boundary conditions needed (Do not solve state only)



Cas II Cas I

Q4) For the circular reinforced concrete tank having the following data:

H= 8 m C (shrinkage strain) =300x10^{-6} f_s (flexural) = 150 MPa base : hinge
D (inside) =32 m t (wall thickness) = 0.5m (to be check) $f_c' = 25$ MPa top : free
γ_w =10 kN/m^3 f_s (hoop) = 100 MPa f_t (concrete) = 2.5 MPa

a- Hoop reinforcement at the critical section (use ∅ 25 bars)

b- Flexural reinforcement at the critical section (use ∅ 16 bars)

Q5) For the rectangular concrete tank having the following data:

Base: hinge a= 6m b= 9m c= 4.5m γ_w =10 kN/m^3
Top: hinge $f_c' = 25$ MPa $f_s = 150$ MPa t=500 mm (to be check)
Find:

a- Vertical reinforcement at the critical section of long wall (use ∅16 bars).

b- Horizontal reinforcement at critical section in the inner face (use ∅16 bars).

Q6) In which case:

a –Distribution reinforcement used in the concrete deck (slab) of bridges?

b-Beam on elastic foundation analysis ?

c-Water stop used in water tanks made of reinforced concrete ?

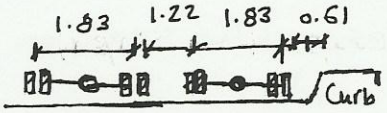
Good Luck

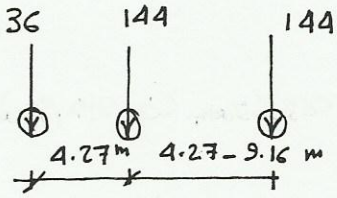
Calculation Sheet	Job Number	WBS Number	Page Number	Sheet of
			90	

Rev	Date	By	Ck	Title

Information sheet

bridges: MS 18 – 44 axle load (kN)





effective span = span c/c ≤ clear span + h

$I = \frac{15.24}{l + 38.1} \leq 0.3$

main reinf. ⊥ traffic

$M_l = \frac{3.28S + 2}{32} P_{18}$ $\qquad A_{sd} = \frac{2.2\,A_s}{\sqrt{3.28S}} \leq 0.67\,A_s$

where $0.6\,m < S < 7.3\,m$

main reinf. // Traffic

$A_{sd} = \frac{A_s}{\sqrt{3.28S}} \leq 0.5\,A_s$ $\qquad E = 1.22 + 0.06S \leq 2.14\,m$

M_l (longitudinal edge beam $= 0.1\,P\,S$

dist. of wheel load Floor system	dist. factor (one – lane)	dist. factor (two or more lanes)
concrete slab on concrete girder	$0.55S$ ($S_{max.} = 2m$)	$0.66S$ ($S_{max.} = 3m$)
concrete slab on steel I – beam or concrete prestressed girders	$0.47S$ ($S_{max.} = 3m$)	$0.6S$ ($S_{max.} = 4.5\,m$)

allowable stresses

$f_c = 0.4\,f_c'$

$f_s = 140$ MPa for $f_y \leq 350$ MPa

170 MPa for $f_y \geq 350$ MPa

$n = \frac{E_s}{E_c}$ $\qquad E_s = 200\,000$ MPa, $E_c = 4730\sqrt{f_c'}$

$r = \frac{f_s}{f_c}$ $\qquad k = \frac{n}{n+r}$ $\qquad j = 1 - \frac{k}{3}$

$d_{req'd} = \sqrt{\frac{2M}{f_c\,k\,j\,b}}$

$A_s = \frac{M}{f_s\,j\,d}$

for R/C T – section $A_s \simeq \frac{M}{f_s\left(d - \frac{h_f}{2}\right)}$

$V_c = 0.079\sqrt{f_c'}\,b_w\,d$

$S = \min.\left\{\frac{d}{2}, 600, \frac{A_v f_s d}{V - V_c}, \frac{3 A_v f_y}{b_w}\right\}$

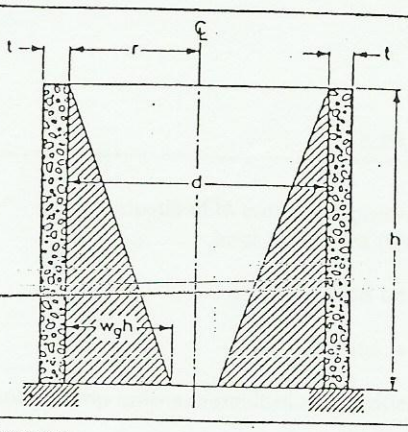
Table A3.1

Ring tension F_t in circular tanks under triangular load.

Fixed base, free top

f_t = coeff × $w_g h r$

Positive sign indicates tension



$\frac{h^2}{dt}$	Coefficients at point from top									
	0.0 h	0.1 h	0.2 h	0.3 h	0.4 h	0.5 h	0.6 h	0.7 h	0.8 h	0.9 h
0.4	+0.149	+0.134	+0.120	+0.101	+0.082	+0.066	+0.049	+0.029	+0.014	+0.004
0.8	+0.263	+0.239	+0.215	+0.190	+0.160	+0.130	+0.096	+0.063	+0.034	+0.010
1.2	+0.283	+0.271	+0.254	+0.234	+0.209	+0.180	+0.142	+0.099	+0.054	+0.016
1.6	+0.265	+0.268	+0.268	+0.266	+0.250	+0.226	+0.185	+0.134	+0.075	+0.023
2.0	+0.234	+0.251	+0.273	+0.285	+0.285	+0.274	+0.232	+0.172	+0.104	+0.031
3.0	+0.134	+0.203	+0.267	+0.322	+0.357	+0.362	+0.330	+0.262	+0.157	+0.052
4.0	+0.067	+0.164	+0.256	+0.339	+0.403	+0.429	+0.409	+0.334	+0.210	+0.073
5.0	+0.025	+0.137	+0.245	+0.346	+0.428	+0.477	+0.469	+0.398	+0.259	+0.092
6.0	+0.018	+0.119	+0.234	+0.344	+0.441	+0.504	+0.514	+0.447	+0.301	+0.112
8.0	−0.011	+0.104	+0.218	+0.335	+0.443	+0.534	+0.575	+0.530	+0.381	+0.151
10.0	−0.011	+0.098	+0.208	+0.323	+0.437	+0.542	+0.608	+0.589	+0.440	+0.179
12.0	−0.005	+0.097	+0.202	+0.312	+0.429	+0.543	+0.628	+0.633	+0.494	+0.211
14.0	−0.002	+0.098	+0.200	+0.306	+0.420	+0.539	+0.639	+0.666	+0.541	+0.241
16.0	0.000	+0.099	+0.199	+0.304	+0.412	+0.531	+0.641	+0.687	+0.582	+0.265

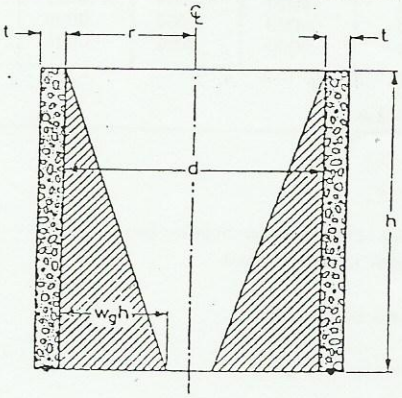
Table A3.2

Ring tension F_t in circular tanks under triangular load.

Hinged base, free top

f_t = coeff × $w_g h r$

Positive sign indicates tension



$\frac{h^2}{dt}$	Coefficients at point from top									
	0.0 h	0.1 h	0.2 h	0.3 h	0.4 h	0.5 h	0.6 h	0.7 h	0.8 h	0.9 h
0.4	+0.474	+0.440	+0.395	+0.352	+0.308	+0.264	+0.215	+0.165	+0.111	+0.057
0.8	+0.423	+0.402	+0.381	+0.358	+0.330	+0.297	+0.249	+0.202	+0.145	+0.076
1.2	+0.350	+0.355	+0.361	+0.362	+0.358	+0.343	+0.309	+0.256	+0.186	+0.098
1.6	+0.271	+0.303	+0.341	+0.369	+0.385	+0.385	+0.362	+0.314	+0.233	+0.124
2.0	+0.205	+0.260	+0.321	+0.373	+0.411	+0.434	+0.419	+0.369	+0.280	+0.151
3.0	+0.074	+0.179	+0.281	+0.375	+0.449	+0.506	+0.519	+0.479	+0.375	+0.210
4.0	+0.017	+0.137	+0.253	+0.367	+0.469	+0.545	+0.579	+0.553	+0.447	+0.256
5.0	−0.008	+0.114	+0.235	+0.356	+0.459	+0.562	+0.617	+0.606	+0.503	+0.294
6.0	−0.011	+0.103	+0.223	+0.343	+0.453	+0.566	+0.639	+0.643	+0.547	+0.327
8.0	−0.015	+0.096	+0.208	+0.324	+0.443	+0.564	+0.661	+0.697	+0.621	+0.386
10.0	−0.008	+0.095	+0.200	+0.311	+0.428	+0.552	+0.663	+0.730	+0.678	+0.433
12.0	−0.002	+0.097	+0.197	+0.302	+0.417	+0.541	+0.654	+0.739	+0.720	+0.477
14.0	0.000	+0.098	+0.197	+0.299	+0.409	+0.531	+0.639	+0.751	+0.752	+0.513
16.0	+0.002	+0.100	+0.198	+0.299	+0.403	+0.521	+0.650	+0.764	+0.775	+0.536

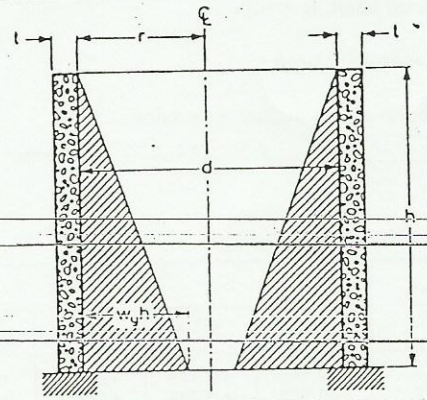
Table A3.3

Bending moments M in circular tanks under triangular load.

Fixed base, free top

m = coeff × $w_g h^3$

Positive sign indicates tension on the outside



$\frac{h^2}{dt}$	Coefficients at point from top									
	0.1 h	0.2 h	0.3 h	0.4 h	0.5 h	0.6 h	0.7 h	0.8 h	0.9 h	1.0 h
0.4	+.0005	+.0014	+.0021	+.0007	−.0042	−.0150	−.0302	−.0529	−.0816	−.1205
0.8	+.0011	+.0037	+.0063	+.0080	+.0070	+.0023	−.0068	−.0224	−.0465	−.0795
1.2	+.0012	+.0042	+.0077	+.0103	+.0112	+.0090	+.0022	−.0108	−.0311	−.0602
1.6	+.0011	+.0041	+.0075	+.0107	+.0121	+.0111	+.0058	−.0051	−.0232	−.0505
2.0	+.0010	+.0035	+.0068	+.0099	+.0120	+.0115	+.0075	−.0021	−.0185	−.0436
3.0	+.0006	+.0024	+.0047	+.0071	+.0090	+.0097	+.0077	+.0012	−.0119	−.0333
4.0	+.0003	+.0015	+.0028	+.0047	+.0066	+.0077	+.0069	+.0023	−.0080	−.0268
5.0	+.0002	+.0008	+.0016	+.0029	+.0046	+.0059	+.0059	+.0028	−.0058	−.0222
6.0	+.0001	+.0003	+.0008	+.0019	+.0032	+.0046	+.0051	+.0029	−.0041	−.0187
8.0	.0000	+.0001	+.0002	+.0008	+.0016	+.0028	+.0038	+.0029	−.0022	−.0146
10.0	.0000	.0000	+.0001	+.0004	+.0007	+.0019	+.0029	+.0028	−.0012	−.0122
12.0	.0000	−.0001	+.0001	+.0002	+.0003	+.0013	+.0023	+.0026	−.0005	−.0104
14.0	.0000	.0000	.0000	.0000	+.0001	+.0008	+.0019	+.0023	−.0001	−.0090
16.0	.0000	.0000	−.0001	−.0002	−.0001	+.0004	+.0013	+.0019	+.0001	−.0079

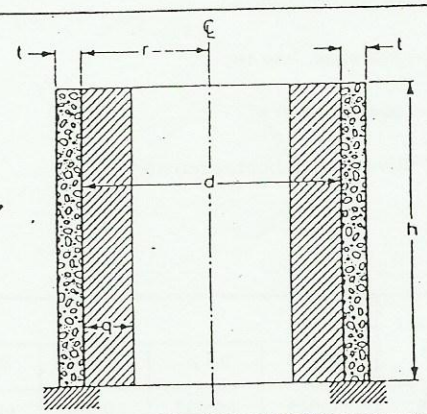
Table A3.4

Ring tension F_t in circular tanks under uniform load.

Fixed base, free top

f_t = coeff × qr

Positive sign indicates tension



$\frac{h^2}{dt}$	Coefficients at point from top									
	0.0 h	0.1 h	0.2 h	0.3 h	0.4 h	0.5 h	0.6 h	0.7 h	0.8 h	0.9 h
0.4	+0.582	+0.505	+0.431	+0.353	+0.277	+0.206	+0.145	+0.092	+0.046	+0.013
0.8	+1.052	+0.921	+0.796	+0.669	+0.542	+0.415	+0.289	+0.179	+0.089	+0.024
1.2	+1.218	+1.078	+0.946	+0.808	+0.665	+0.519	+0.378	+0.246	+0.127	+0.034
1.6	+1.257	+1.141	+1.009	+0.881	+0.742	+0.600	+0.449	+0.294	+0.153	+0.045
2.0	+1.253	+1.144	+1.041	+0.929	+0.806	+0.667	+0.514	+0.345	+0.186	+0.055
3.0	+1.160	+1.112	+1.061	+0.998	+0.912	+0.796	+0.646	+0.459	+0.258	+0.081
4.0	+1.085	+1.073	+1.057	+1.029	+0.977	+0.887	+0.746	+0.553	+0.322	+0.105
5.0	+1.037	+1.044	+1.047	+1.042	+1.015	+0.949	+0.825	+0.629	+0.379	+0.128
6.0	+1.010	+1.024	+1.038	+1.045	+1.034	+0.986	+0.879	+0.694	+0.430	+0.149
8.0	+0.989	+1.005	+1.022	+1.036	+1.044	+1.026	+0.953	+0.788	+0.519	+0.189
10.0	+0.989	+0.998	+1.010	+1.023	+1.039	+1.040	+0.996	+0.859	+0.591	+0.226
12.0	+0.994	+0.997	+1.003	+1.014	+1.031	+1.043	+1.022	+0.911	+0.652	+0.262
14.0	+0.997	+0.998	+1.000	+1.007	+1.022	+1.040	+1.035	+0.949	+0.705	+0.294
16.0	+1.000	+0.999	+0.999	+1.003	+1.015	+1.032	+1.040	+0.975	+0.750	+0.321

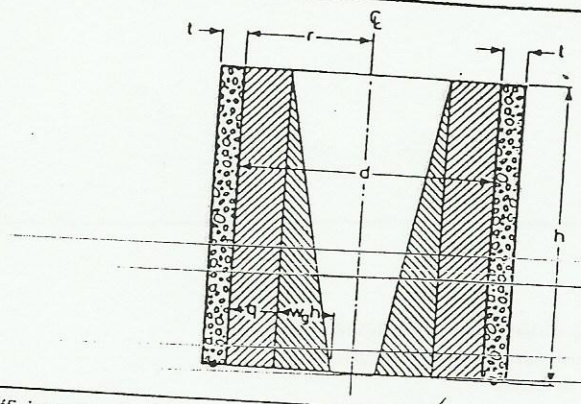
Table A3.7

Bending moment M in circular tanks under combined triangular and uniform load.

Hinged base, free top

m = coeff × $(w_g h^3 + qh^2)$

Positive sign indicates tension on the outside



$\frac{h^2}{dt}$	Coefficients at point from top									
	0.1 h	0.2 h	0.3 h	0.4 h	0.5 h	0.6 h	0.7 h	0.8 h	0.9 h	1.0 h
0.4	+.0020	+.0072	+.0151	+.0230	+.0301	+.0348	+.0357	+.0312	+.0197	0
0.8	+.0019	+.0064	+.0133	+.0207	+.0271	+.0319	+.0329	+.0292	+.0187	0
1.2	+.0016	+.0058	+.0111	+.0177	+.0237	+.0280	+.0296	+.0263	+.0171	0
1.6	+.0012	+.0044	+.0091	+.0145	+.0195	+.0236	+.0255	+.0232	+.0155	0
2.0	+.0009	+.0033	+.0073	+.0114	+.0158	+.0199	+.0219	+.0205	+.0145	0
3.0	+.0004	+.0018	+.0040	+.0063	+.0092	+.0127	+.0152	+.0153	+.0111	0
4.0	+.0001	+.0007	+.0016	+.0033	+.0057	+.0083	+.0109	+.0118	+.0092	0
5.0	.0000	+.0001	+.0006	+.0016	+.0034	+.0057	+.0080	+.0094	+.0078	0
6.0	.0000	.0000	+.0002	+.0008	+.0019	+.0039	+.0062	+.0078	+.0068	0
8.0	.0000	.0000	−.0002	.0000	+.0007	+.0020	+.0038	+.0057	+.0054	0
10.0	.0000	.0000	−.0002	−.0001	+.0002	+.0011	+.0025	+.0043	+.0045	0
12.0	.0000	.0000	−.0001	−.0002	.0000	+.0005	+.0017	+.0032	+.0039	0
14.0	.0000	.0000	−.0001	−.0001	−.0001	.0000	+.0012	+.0026	+.0033	0
16.0	.0000	.0000	.0000	−.0001	−.0002	−.0004	+.0008	+.0022	+.0029	0

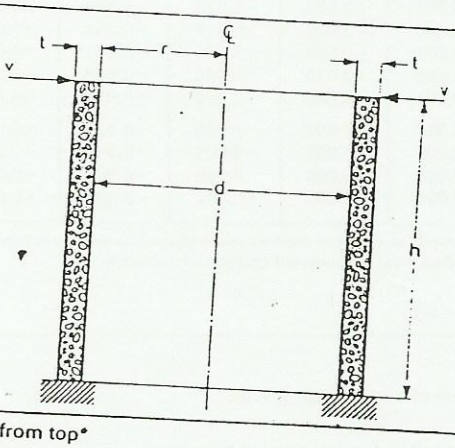
Table A3.8

Ring tension F_t in circular tanks under shear per unit length v applied at top.

Fixed base, free top

f_t = coeff × $\frac{vr}{h}$

Positive sign indicates tension



$\frac{h^2}{dt}$	Coefficients at point from top*									
	0.0 h	0.1 h	0.2 h	0.3 h	0.4 h	0.5 h	0.6 hc	0.7 $h1$	0.8 h	0.9 h
0.4	− 1.57	−1.32	−1.08	−0.86	−0.65	−0.47	−0.31	−0.18	−0.08	−0.02
0.8	− 3.09	−2.55	−2.04	−1.57	−1.15	−0.80	−0.51	−0.28	−0.13	−0.03
1.2	− 3.95	−3.17	−2.44	−1.79	−1.25	−0.81	−0.48	−0.25	−0.10	−0.02
1.6	− 4.57	−3.54	−2.60	−1.80	−1.17	−0.69	−0.36	−0.16	−0.05	−0.01
2.0	− 5.12	−3.83	−2.68	−1.74	−1.02	−0.52	−0.21	−0.05	+0.01	+0.01
3.0	− 6.32	−4.37	−2.70	−1.43	−0.58	−0.02	+0.15	+0.19	+0.13	+0.04
4.0	− 7.34	−4.73	−2.60	−1.10	−0.19	+0.26	+0.38	+0.33	+0.19	+0.06
5.0	− 8.22	−4.99	−2.45	−0.79	+0.11	+0.47	+0.50	+0.37	+0.20	+0.06
6.0	− 9.02	−5.17	−2.27	−0.50	+0.34	+0.59	+0.53	+0.35	+0.17	+0.01
8.0	−10.42	−5.36	−1.85	−0.02	+0.63	+0.66	+0.46	+0.24	+0.09	+0.01
10.0	−11.67	−5.43	−1.43	+0.36	+0.78	+0.62	+0.33	+0.12	+0.02	0.00
12.0	−12.76	−5.41	−1.03	+0.63	+0.83	+0.52	+0.21	+0.04	−0.02	0.00
14.0	−13.77	−5.34	−0.68	+0.80	+0.81	+0.42	+0.13	0.00	−0.03	−0.01
16.0	−14.74	−5.22	−0.33	+0.96	+0.76	+0.32	+0.05	+0.04	−0.05	−0.02

*When this Table is used for shear applied at the base, while the top is fixed, 0.0 h is the bottom of the wall and 1.0 h is the top. Shear acting inward is positive, outward is negative.

Table A4.1

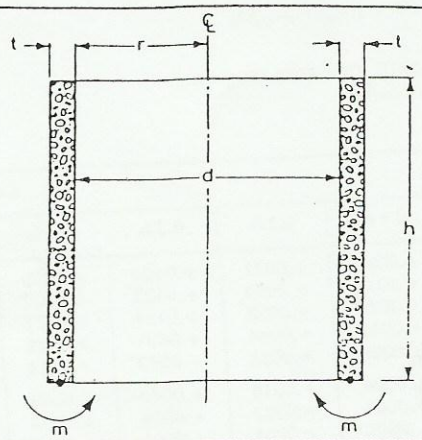
Table A3.11

Bending moments M in circular tanks under moment per unit length m applied at base.

Hinged base, free top

M = coeff × m

Positive sign indicates tension on the outside



$\frac{h^2}{dt}$	Coefficients at point from top*									
	0.1 h	0.2 h	0.3 h	0.4 h	0.5 h	0.6 h	0.7 h	0.8 h	0.9 h	1.0 h
0.4	+0.013	+0.051	+0.109	+0.196	+0.296	+0.414	+0.547	+0.692	+0.843	+1.000
0.8	+0.009	+0.040	+0.090	+0.164	+0.253	+0.375	+0.503	+0.659	+0.824	+1.000
1.2	+0.006	+0.027	+0.063	+0.125	+0.206	+0.316	+0.454	+0.616	+0.802	+1.000
1.6	+0.003	+0.011	+0.035	+0.078	+0.152	+0.253	+0.393	+0.570	+0.775	+1.000
2.0	−0.002	−0.002	+0.012	+0.034	+0.096	+0.193	+0.340	+0.519	+0.748	+1.000
3.0	−0.007	−0.022	−0.030	−0.029	+0.010	+0.087	+0.227	+0.426	+0.692	+1.000
4.0	−0.006	−0.026	−0.044	−0.051	−0.034	+0.023	+0.150	+0.354	+0.645	+1.000
5.0	−0.007	−0.024	−0.045	−0.061	−0.057	−0.015	+0.095	+0.296	+0.606	+1.000
6.0	−0.005	−0.018	−0.040	−0.058	−0.065	−0.037	+0.057	+0.252	+0.572	+1.000
8.0	−0.001	−0.009	−0.022	−0.044	−0.068	−0.062	+0.002	+0.178	+0.515	+1.000
10.0	0.000	−0.002	−0.009	−0.028	−0.053	−0.067	−0.031	+0.123	+0.467	+1.000
12.0	0.000	0.000	−0.003	−0.016	−0.040	−0.064	−0.049	+0.081	+0.424	+1.000
14.0	0.000	0.000	0.000	−0.008	−0.029	−0.059	−0.060	+0.048	+0.387	+1.000
16.0	0.000	0.000	+0.002	−0.003	−0.021	−0.051	−0.066	+0.025	+0.354	+1.000

*When this Table is used for moment applied at the top, while the top is hinged, 0.0 h is the bottom of the wall and 1.0 h is the top. Moment applied at an edge is positive when it causes outward rotation at that edge.

Table A3.12

Shear at base of circular tank wall

v = coeff × $w_g h^2$ · for triangular loading

× qh for uniform loading

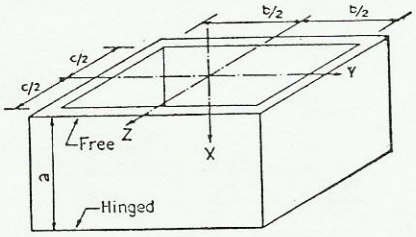
× $\frac{m}{h}$ for moment at base

Positive sign indicates shear acting inward

$\frac{h^2}{dt}$	Triangular load fixed base	Uniform load fixed base	Triangular or uniform load, hinged base	Moment at base
0.4	+0.436	+0.755	+0.245	−1.58
0.8	+0.374	+0.552	+0.234	−1.75
1.2	+0.339	+0.460	+0.220	−2.00
1.6	+0.317	+0.407	+0.204	−2.28
2.0	+0.299	+0.370	+0.189	−2.57
3.0	+0.262	+0.310	+0.158	−3.18
4.0	+0.236	+0.271	+0.137	−3.68
5.0	+0.213	+0.243	+0.121	−4.10
6.0	+0.197	+0.222	+0.110	−4.49
8.0	+0.174	+0.193	+0.096	−5.18
10.0	+0.158	+0.172	+0.087	−5.81
12.0	+0.145	+0.158	+0.079	−6.38
14.0	+0.135	+0.147	+0.073	−6.88

Table V. Moment Coefficients for Tanks with Walls Free at Top and Hinged at Bottom

Moment = Coef. × wa^3



b/a = 3.0

c/a	x/a	y = 0		y = b/4		y = b/2		z = c/4		z = 0	
		M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_z	M_x	M_z
3.00	0	0	+.070	0	+.027	0	-.196	0	+.027	0	+.070
	¼	+.028	+.061	+.015	+.028	-.034	-.170	+.015	+.028	+.028	+.061
	½	+.049	+.049	+.032	+.026	-.027	-.137	+.032	+.026	+.049	+.049
	¾	+.046	+.030	+.034	+.018	-.017	-.087	+.034	+.018	+.046	+.030
2.50	0	0	+.073	0	+.033	0	-.169	0	+.013	0	+.057
	¼	+.028	+.063	+.016	+.033	-.030	-.151	+.009	+.014	+.022	+.050
	½	+.049	+.050	+.033	+.029	-.025	-.126	+.023	+.017	+.041	+.043
	¾	+.046	+.030	+.037	+.020	-.017	-.084	+.029	+.014	+.040	+.027
2.00	0	0	+.075	0	+.039	0	-.146	0	-.005	0	+.031
	¼	+.029	+.065	+.017	+.036	-.027	-.133	+.002	-.002	+.013	+.032
	½	+.050	+.051	+.035	+.032	-.023	-.113	+.016	+.005	+.030	+.029
	¾	+.046	+.031	+.037	+.021	-.016	-.078	+.022	+.008	+.034	+.020
1.75	0	0	+.076	0	+.041	0	-.137	0	-.018	0	+.014
	¼	+.029	+.065	+.018	+.038	-.025	-.125	-.003	-.012	+.007	+.018
	½	+.050	+.052	+.036	+.033	-.021	-.106	+.011	-.003	+.023	+.020
	¾	+.046	+.031	+.037	+.021	-.015	-.074	+.018	+.004	+.027	+.015
1.50	0	0	+.077	0	+.043	0	-.129	0	-.033	0	-.006
	¼	+.030	+.066	+.018	+.039	-.024	-.118	-.007	-.024	+.002	+.004
	½	+.050	+.053	+.037	+.034	-.020	-.100	+.005	-.012	+.015	+.010
	¾	+.046	+.031	+.038	+.022	-.014	-.070	+.013	-.000	+.021	+.010
1.25	0	0	+.078	0	+.045	0	-.122	0	-.052	0	-.031
	¼	+.030	+.067	+.019	+.041	-.022	-.111	-.011	-.039	-.004	-.018
	½	+.050	+.054	+.038	+.035	-.019	-.095	-.000	-.022	+.008	-.005
	¾	+.047	+.032	+.038	+.023	-.014	-.068	+.008	-.006	+.016	+.001
1.00	0	0	+.079	0	+.047	0	-.118	0	-.074	0	-.060
	¼	+.030	+.067	+.020	+.043	-.021	-.105	-.015	-.056	-.010	-.042
	½	+.051	+.054	+.038	+.036	-.018	-.090	-.005	-.034	+.001	-.022
	¾	+.047	+.032	+.038	+.023	-.013	-.065	+.003	-.014	+.009	-.009
0.75	0	0	+.079	0	+.047	0	-.120	0	-.098	0	-.092
	¼	+.029	+.066	+.020	+.042	-.021	-.107	-.020	-.079	-.016	-.070
	½	+.051	+.053	+.037	+.036	-.018	-.090	-.011	-.051	-.006	-.045
	¾	+.047	+.031	+.037	+.022	-.013	-.066	-.002	-.025	+.003	-.024
0.50	0	0	+.078	0	+.047	0	-.130	0	-.126	0	-.123
	¼	+.029	+.065	+.019	+.042	-.023	-.115	-.024	-.105	-.022	-.101
	½	+.050	+.053	+.035	+.035	-.019	-.095	-.016	-.073	-.013	-.071
	¾	+.046	+.031	+.036	+.021	-.014	-.068	-.007	-.040	-.004	-.042

b/a = 2.0

c/a	x/a	y = 0		y = b/4		y = b/2		z = c/4		z = 0	
		M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_z	M_x	M_z
2.00	0	0	+.045	0	+.011	0	-.091	0	+.011	0	+.045
	¼	+.016	+.042	+.006	+.014	-.019	-.094	+.006	+.014	+.016	+.042
	½	+.033	+.036	+.020	+.016	-.018	-.089	+.020	+.016	+.033	+.036
	¾	+.036	+.024	+.025	+.014	-.013	-.065	+.025	+.014	+.036	+.024
1.75	0	0	+.048	0	+.015	0	-.081	0	-.001	0	+.032
	¼	+.017	+.044	+.007	+.017	-.017	-.085	+.003	+.006	+.012	+.032
	½	+.034	+.038	+.021	+.019	-.017	-.083	+.015	+.011	+.027	+.029
	¾	+.036	+.024	+.025	+.015	-.012	-.061	+.020	+.012	+.031	+.021
1.50	0	0	+.050	0	+.019	0	-.072	0	-.010	0	+.018
	¼	+.018	+.046	+.008	+.021	-.015	-.077	+.000	-.002	+.007	+.020
	½	+.035	+.039	+.022	+.021	-.015	-.076	+.009	+.004	+.020	+.022
	¾	+.036	+.025	+.026	+.016	-.012	-.058	+.016	+.008	+.025	+.017
1.25	0	0	+.052	0	+.023	0	-.064	0	-.021	0	-.000
	¼	+.019	+.048	+.009	+.024	-.014	-.068	-.002	-.013	+.001	+.005
	½	+.036	+.041	+.023	+.023	-.014	-.069	+.005	-.004	+.011	+.012
	¾	+.037	+.025	+.026	+.017	-.011	-.054	+.011	+.002	+.016	+.011
1.00	0	0	+.054	0	+.027	0	-.058	0	-.037	0	-.023
	¼	+.019	+.050	+.010	+.027	-.012	-.062	-.005	-.025	-.005	-.013
	½	+.037	+.042	+.024	+.025	-.013	-.064	-.000	-.015	+.001	+.000
	¾	+.037	+.026	+.027	+.018	-.010	-.051	+.006	-.006	+.008	+.004
0.75	0	0	+.055	0	+.030	0	-.058	0	-.049	0	-.044
	¼	+.018	+.051	+.011	+.029	-.012	-.062	-.009	-.040	-.010	-.031
	½	+.038	+.043	+.025	+.026	-.012	-.062	-.005	-.029	-.007	-.015
	¾	+.037	+.026	+.027	+.018	-.010	-.049	+.002	-.015	+.001	-.004
0.50	0	0	+.054	0	+.030	0	-.065	0	-.064	0	-.061
	¼	+.018	+.052	+.011	+.029	-.014	-.068	-.012	-.056	-.014	-.051
	½	+.038	+.044	+.025	+.025	-.013	-.064	-.010	-.045	-.012	-.034
	¾	+.037	+.026	+.026	+.017	-.010	-.050	-.003	-.026	-.004	-.018

b/a = 2.5

c/a	x/a	y = 0		y = b/4		y = b/2		z = c/4		z = 0	
		M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_z	M_x	M_z
2.50	0	0	+.061	0	+.019	0	-.138	0	+.019	0	+.061
	¼	+.024	+.053	+.010	+.022	-.026	-.132	+.010	+.022	+.024	+.053
	½	+.042	+.044	+.025	+.022	-.023	-.115	+.025	+.022	+.042	+.044
	¾	+.041	+.027	+.030	+.016	-.016	-.078	+.030	+.016	+.041	+.027
2.00	0	0	+.065	0	+.026	0	-.118	0	+.003	0	+.038
	¼	+.025	+.055	+.012	+.027	-.023	-.113	+.005	+.006	+.015	+.037
	½	+.043	+.046	+.028	+.025	-.020	-.102	+.018	+.011	+.032	+.033
	¾	+.042	+.028	+.031	+.018	-.014	-.070	+.023	+.011	+.034	+.022
1.75	0	0	+.067	0	+.030	0	-.108	0	-.006	0	+.025
	¼	+.025	+.057	+.013	+.030	-.021	-.104	+.001	-.002	+.010	+.026
	½	+.044	+.047	+.029	+.027	-.019	-.096	+.013	+.004	+.025	+.025
	¾	+.043	+.028	+.033	+.019	-.013	-.066	+.019	+.008	+.029	+.019
1.50	0	0	+.068	0	+.033	0	-.100	0	-.018	0	+.008
	¼	+.026	+.058	+.014	+.032	-.019	-.097	-.003	-.012	+.004	+.013
	½	+.045	+.047	+.030	+.029	-.018	-.089	+.008	-.002	+.017	+.017
	¾	+.043	+.029	+.034	+.019	-.013	-.063	+.015	+.004	+.024	+.015
1.25	0	0	+.069	0	+.035	0	-.092	0	-.030	0	-.010
	¼	+.026	+.059	+.015	+.034	-.018	-.089	-.006	-.024	-.002	-.003
	½	+.045	+.048	+.031	+.031	-.016	-.082	+.003	-.012	+.008	+.007
	¾	+.044	+.029	+.034	+.020	-.012	-.059	+.011	-.002	+.018	+.008
1.00	0	0	+.070	0	+.037	0	-.087	0	-.045	0	-.032
	¼	+.026	+.060	+.015	+.036	-.017	-.083	-.010	-.036	-.008	-.021
	½	+.046	+.048	+.031	+.032	-.015	-.077	-.003	-.021	-.001	-.008
	¾	+.044	+.029	+.033	+.021	-.011	-.056	+.006	-.008	+.011	-.000
0.75	0	0	+.070	0	+.038	0	-.082	0	-.062	0	-.055
	¼	+.025	+.060	+.015	+.037	-.016	-.078	-.014	-.053	-.014	-.042
	½	+.045	+.047	+.030	+.032	-.014	-.071	-.008	-.035	-.009	-.025
	¾	+.043	+.029	+.033	+.020	-.011	-.054	+.002	-.016	+.005	-.011
0.50	0	0	+.069	0	+.039	0	-.080	0	-.081	0	-.080
	¼	+.025	+.059	+.014	+.038	-.015	-.075	-.019	-.072	-.019	-.068
	½	+.044	+.046	+.028	+.032	-.014	-.068	-.014	-.056	-.017	-.048
	¾	+.042	+.028	+.032	+.019	-.010	-.052	-.003	-.030	-.002	-.026

b/a = 1.5

c/a	x/a	y = 0		y = b/4		y = b/2		z = c/4		z = 0	
		M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_z	M_x	M_z
1.50	0	0	+.027	0	+.005	0	-.052	0	+.005	0	+.027
	¼	+.009	+.028	+.003	+.008	-.012	-.059	+.003	+.008	+.009	+.028
	½	+.022	+.027	+.012	+.011	-.013	-.063	+.012	+.011	+.022	+.027
	¾	+.027	+.020	+.017	+.011	-.010	-.052	+.017	+.011	+.027	+.020
1.25	0	0	+.031	0	+.008	0	-.045	0	-.005	0	+.011
	¼	+.010	+.031	+.005	+.012	-.010	-.050	+.001	-.001	+.004	+.015
	½	+.024	+.030	+.014	+.014	-.011	-.056	+.007	+.006	+.014	+.020
	¾	+.027	+.021	+.018	+.012	-.010	-.048	+.013	+.006	+.018	+.016
1.00	0	0	+.035	0	+.013	0	-.038	0	-.016	0	-.006
	¼	+.011	+.034	+.006	+.016	-.008	-.042	-.002	-.010	-.001	+.001
	½	+.025	+.032	+.015	+.017	-.010	-.049	+.002	-.003	+.006	+.010
	¾	+.028	+.022	+.019	+.014	-.009	-.045	+.008	+.002	+.009	+.010
0.75	0	0	+.038	0	+.016	0	-.034	0	-.024	0	-.019
	¼	+.011	+.036	+.007	+.018	-.008	-.038	-.005	-.020	-.004	-.013
	½	+.025	+.033	+.016	+.019	-.008	-.042	-.002	-.014	-.001	-.004
	¾	+.028	+.022	+.019	+.015	-.008	-.041	+.003	-.007	+.002	+.001
0.50	0	0	+.040	0	+.017	0	-.036	0	-.030	0	-.028
	¼	+.010	+.037	+.007	+.019	-.008	-.040	-.008	-.031	-.007	-.027
	½	+.024	+.034	+.017	+.020	-.009	-.044	-.006	-.027	-.006	-.020
	¾	+.028	+.022	+.018	+.016	-.008	-.040	-.002	-.018	-.004	-.010

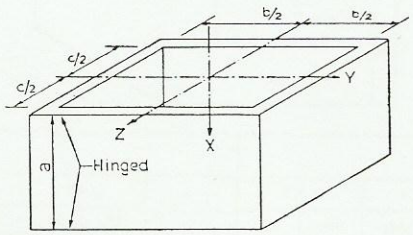
b/a = 1.0

c/a	x/a	y = 0		y = b/4		y = b/2		z = c/4		z = 0	
		M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_z	M_x	M_z
1.00	0	0	+.010	0	+.002	0	-.019	0	+.002	0	+.010
	¼	+.002	+.013	+.000	+.003	-.005	-.025	+.000	+.003	+.002	+.013
	½	+.010	+.017	+.005	+.006	-.007	-.036	+.005	+.006	+.010	+.017
	¾	+.015	+.015	+.009	+.007	-.007	-.036	+.009	+.007	+.015	+.015
0.75	0	0	+.016	0	+.007	0	-.013	0	-.004	0	+.003
	¼	+.003	+.017	+.001	+.008	-.004	-.020	-.001	-.005	-.001	+.003
	½	+.011	+.020	+.006	+.009	-.007	-.033	+.002	-.001	+.005	+.007
	¾	+.016	+.014	+.009	+.009	-.006	-.032	+.004	+.002	+.009	+.008
0.50	0	0	+.020	0	+.011	0	-.011	0	-.007	0	-.005
	¼	+.003	+.018	+.001	+.010	-.004	-.018	-.002	-.012	-.003	-.007
	½	+.012	+.021	+.008	+.010	-.006	-.032	+.001	-.009	+.002	-.005
	¾	+.017	+.013	+.010	+.009	-.006	-.031	+.002	-.005	+.006	+.001

Minus sign indicates tension on the loaded side.

Table VI. Moment Coefficients for Tanks with Walls Hinged at Top and Bottom

Moment = Coef. × wa^3



b/a = 3.0

c/a	x/a	y = 0		y = b/4		y = b/2		z = c/4		z = 0	
		M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_z	M_x	M_z
3.00	¼	+.035	+.010	+.026	+.011	-.008	-.039	+.026	+.011	+.035	+.010
	½	+.057	+.016	+.044	+.017	-.013	-.063	+.044	+.017	+.057	+.016
	¾	+.051	+.013	+.041	+.014	-.011	-.055	+.041	+.014	+.051	+.013
2.50	¼	+.035	+.010	+.026	+.011	-.008	-.039	+.021	+.010	+.031	+.011
	½	+.057	+.016	+.044	+.017	-.012	-.062	+.036	+.017	+.052	+.017
	¾	+.051	+.013	+.041	+.014	-.011	-.055	+.036	+.014	+.047	+.014
2.00	¼	+.035	+.010	+.026	+.011	-.008	-.038	+.015	+.010	+.025	+.013
	½	+.057	+.016	+.045	+.017	-.012	-.062	+.028	+.015	+.043	+.020
	¾	+.051	+.013	+.042	+.014	-.011	-.054	+.029	+.013	+.041	+.016
1.75	¼	+.035	+.010	+.027	+.011	-.007	-.037	+.011	+.008	+.020	+.013
	½	+.057	+.015	+.045	+.017	-.012	-.060	+.021	+.013	+.036	+.020
	¾	+.051	+.013	+.042	+.014	-.011	-.053	+.024	+.012	+.036	+.016
1.50	¼	+.035	+.010	+.027	+.011	-.007	+.035	+.007	+.006	+.014	+.013
	½	+.057	+.015	+.045	+.017	-.011	-.057	+.015	+.010	+.027	+.020
	¾	+.051	+.013	+.042	+.014	-.010	-.051	+.019	+.011	+.029	+.017
1.25	¼	+.035	+.010	+.027	+.011	-.006	-.032	+.003	+.003	+.008	+.011
	½	+.057	+.015	+.046	+.017	-.011	-.053	+.008	+.006	+.017	+.017
	¾	+.051	+.013	+.042	+.014	-.010	-.048	+.013	+.008	+.021	+.016
1.00	¼	+.035	+.010	+.027	+.011	-.006	-.029	-.001	+.000	+.002	+.008
	½	+.057	+.015	+.046	+.017	-.010	-.048	+.002	+.002	+.007	+.014
	¾	+.051	+.013	+.043	+.014	-.009	-.044	+.007	+.004	+.013	+.013
0.75	¼	+.035	+.010	+.028	+.011	-.005	-.025	-.003	-.005	-.002	+.001
	½	+.057	+.015	+.046	+.017	-.008	-.042	-.003	-.005	-.001	+.007
	¾	+.052	+.013	+.043	+.014	-.008	-.039	+.002	-.002	+.006	+.007
0.50	¼	+.036	+.010	+.028	+.011	-.004	-.021	-.004	-.011	-.005	-.008
	½	+.057	+.015	+.047	+.017	-.007	-.035	-.007	-.016	-.006	-.010
	¾	+.052	+.013	+.043	+.014	-.007	-.033	-.004	-.010	-.001	-.004

b/a = 2.5

c/a	x/a	y = 0		y = b/4		y = b/2		z = c/4		z = 0	
		M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_z	M_x	M_z
2.50	¼	+.031	+.011	+.021	+.010	-.008	-.038	+.021	+.010	+.031	+.011
	½	+.052	+.017	+.036	+.017	-.012	-.062	+.036	+.017	+.052	+.017
	¾	+.047	+.015	+.036	+.014	-.011	-.055	+.036	+.014	+.047	+.015
2.00	¼	+.031	+.011	+.021	+.010	-.008	-.038	+.015	+.009	+.025	+.012
	½	+.052	+.017	+.036	+.017	-.012	-.061	+.028	+.015	+.042	+.020
	¾	+.047	+.015	+.036	+.014	-.011	-.054	+.029	+.013	+.041	+.016
1.75	¼	+.032	+.011	+.021	+.010	-.007	-.037	+.011	+.008	+.020	+.012
	½	+.052	+.018	+.036	+.017	-.012	-.059	+.022	+.013	+.035	+.021
	¾	+.047	+.015	+.036	+.014	-.011	-.053	+.024	+.012	+.035	+.017
1.50	¼	+.032	+.011	+.022	+.010	-.007	-.035	+.007	+.006	+.014	+.013
	½	+.052	+.018	+.037	+.017	-.011	-.057	+.015	+.010	+.027	+.021
	¾	+.047	+.015	+.036	+.014	-.010	-.051	+.019	+.010	+.029	+.017
1.25	¼	+.032	+.011	+.022	+.010	-.006	-.032	+.003	+.004	+.007	+.012
	½	+.052	+.018	+.038	+.017	-.011	-.053	+.008	+.007	+.018	+.019
	¾	+.048	+.015	+.037	+.014	-.010	-.048	+.014	+.008	+.022	+.016
1.00	¼	+.032	+.011	+.023	+.011	-.006	-.028	-.001	+.000	+.002	+.008
	½	+.053	+.018	+.038	+.017	-.010	-.048	+.002	+.002	+.007	+.014
	¾	+.048	+.015	+.038	+.015	-.009	-.044	+.007	+.004	+.013	+.013
0.75	¼	+.033	+.011	+.024	+.011	-.005	-.024	-.003	-.005	-.002	+.002
	½	+.054	+.018	+.039	+.017	-.008	-.041	-.003	-.005	-.000	+.005
	¾	+.049	+.015	+.038	+.015	-.008	-.039	+.000	-.002	+.006	+.006
0.50	¼	+.033	+.012	+.024	+.011	-.004	-.021	-.004	-.011	-.005	-.008
	½	+.054	+.018	+.040	+.017	-.007	-.035	-.007	-.016	-.006	-.010
	¾	+.049	+.015	+.039	+.015	-.007	-.034	-.004	-.010	-.001	-.004

b/a = 2.0

c/a	x/a	y = 0		y = b/4		y = b/2		z = c/4		z = 0	
		M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_z	M_x	M_z
2.00	¼	+.025	+.013	+.015	+.009	-.007	-.037	+.015	+.009	+.025	+.013
	½	+.042	+.020	+.028	+.015	-.012	-.059	+.028	+.015	+.042	+.020
	¾	+.040	+.016	+.029	+.013	-.011	-.053	+.029	+.013	+.040	+.016
1.75	¼	+.025	+.013	+.015	+.009	-.007	-.036	+.011	+.008	+.020	+.013
	½	+.042	+.020	+.028	+.015	-.012	-.058	+.022	+.013	+.035	+.021
	¾	+.040	+.016	+.029	+.013	-.010	-.052	+.024	+.012	+.035	+.017
1.50	¼	+.025	+.013	+.016	+.009	-.007	-.034	+.007	+.006	+.014	+.013
	½	+.043	+.020	+.028	+.015	-.011	-.056	+.015	+.011	+.027	+.021
	¾	+.041	+.016	+.029	+.013	-.010	-.050	+.019	+.010	+.029	+.017
1.25	¼	+.026	+.013	+.016	+.010	-.006	-.032	+.003	+.003	+.007	+.011
	½	+.043	+.020	+.029	+.015	-.010	-.052	+.008	+.007	+.018	+.019
	¾	+.041	+.016	+.030	+.013	-.010	-.048	+.013	+.008	+.021	+.016
1.00	¼	+.026	+.013	+.017	+.010	-.006	-.028	-.001	+.000	+.002	+.008
	½	+.044	+.020	+.030	+.016	-.009	-.046	+.002	+.002	+.007	+.014
	¾	+.041	+.016	+.031	+.014	-.009	-.044	+.007	+.004	+.013	+.013
0.75	¼	+.027	+.013	+.018	+.010	-.005	-.024	-.003	-.004	-.001	+.002
	½	+.045	+.020	+.031	+.016	-.008	-.040	-.002	-.004	+.000	+.005
	¾	+.042	+.016	+.032	+.014	-.008	-.041	+.002	-.002	+.005	+.008
0.50	¼	+.027	+.013	+.019	+.010	-.004	-.021	-.004	-.010	-.004	-.007
	½	+.046	+.020	+.033	+.017	-.007	-.034	-.006	-.015	-.006	-.009
	¾	+.042	+.016	+.032	+.015	-.007	-.037	-.003	-.010	-.002	-.003

b/a = 1.5

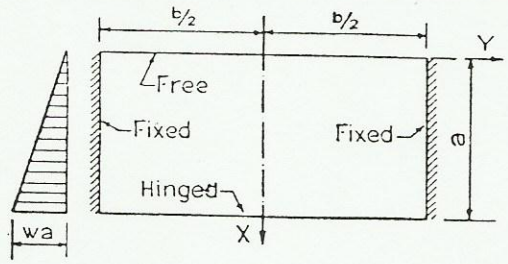
c/a	x/a	y = 0		y = b/4		y = b/2		z = c/4		z = 0	
		M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_z	M_x	M_z
1.50	¼	+.015	+.013	+.008	+.007	-.006	-.032	+.008	+.007	+.015	+.013
	½	+.028	+.021	+.016	+.011	-.010	-.052	+.016	+.011	+.028	+.021
	¾	+.030	+.017	+.020	+.011	-.010	-.048	+.020	+.011	+.030	+.017
1.25	¼	+.016	+.013	+.009	+.008	-.006	-.029	+.004	+.004	+.009	+.012
	½	+.029	+.021	+.017	+.012	-.010	-.049	+.009	+.008	+.018	+.019
	¾	+.030	+.017	+.020	+.012	-.009	-.045	+.014	+.009	+.023	+.016
1.00	¼	+.016	+.013	+.010	+.009	-.005	-.025	+.000	+.001	+.003	+.008
	½	+.030	+.021	+.019	+.012	-.009	-.043	+.003	+.003	+.008	+.014
	¾	+.031	+.017	+.021	+.013	-.008	-.041	+.008	+.005	+.014	+.014
0.75	¼	+.018	+.014	+.011	+.010	-.004	-.021	-.002	-.003	-.001	+.002
	½	+.032	+.022	+.021	+.014	-.007	-.036	-.002	-.004	+.001	+.005
	¾	+.032	+.018	+.022	+.014	-.007	-.036	+.002	-.000	+.006	+.008
0.50	¼	+.020	+.016	+.013	+.012	-.003	-.017	-.003	-.009	-.004	-.006
	½	+.035	+.024	+.023	+.018	-.006	-.031	-.006	-.014	-.005	-.007
	¾	+.034	+.020	+.024	+.016	-.007	-.033	-.003	-.008	-.001	-.001

b/a = 1.0

c/a	x/a	y = 0		y = b/4		y = b/2		z = c/4		z = 0	
		M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_z	M_x	M_z
1.00	¼	+.005	+.009	+.002	+.003	-.004	-.020	+.002	+.003	+.005	+.009
	½	+.011	+.016	+.006	+.006	-.007	-.035	+.006	+.006	+.011	+.016
	¾	+.016	+.015	+.009	+.007	-.007	-.035	+.009	+.007	+.016	+.015
0.75	¼	+.006	+.010	+.003	+.004	-.003	-.016	+.000	+.000	+.001	+.005
	½	+.013	+.017	+.008	+.008	-.006	-.029	+.001	+.001	+.005	+.009
	¾	+.017	+.016	+.010	+.008	-.006	-.031	+.004	+.003	+.008	+.010
0.50	¼	+.007	+.011	+.005	+.006	-.002	-.010	-.002	-.005	-.003	-.002
	½	+.015	+.018	+.010	+.010	-.004	-.021	-.003	-.007	-.003	-.002
	¾	+.018	+.016	+.012	+.010	-.005	-.026	-.001	-.004	-.000	+.001

Minus sign indicates tension on the loaded side.

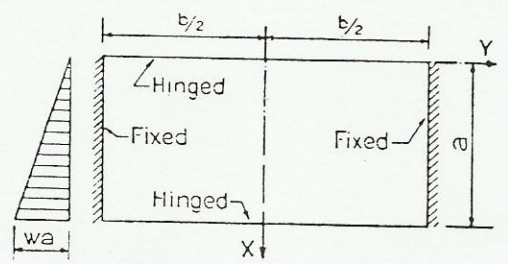
Table VIII. Shear at Edges of Slabs Free at Top and Hinged at Bottom*



b/a	½	1	2	3
Mid-point of bottom edge	$+.141\,wa^2$	$+.242\,wa^2$	$+.38\,wa^2$†	$+.45\,wa^2$†
Corner of bottom edge	$-.258\,wa^2$‡	$-.440\,wa^2$	$-.583\,wa^2$	$-.590\,wa^2$
Top of fixed side edge	$.000\,wa^2$	$+.010\,wa^2$	$+.100\,wa^2$	$+.165\,wa^2$
Mid-point of fixed side edge	$+.128\,wa^2$	$+.258\,wa^2$	$+.375\,wa^2$	$+.406\,wa^2$
Lower third-point of side edge	$+.174\,wa^2$	$+.311\,wa^2$	$+.406\,wa^2$	$+.416\,wa^2$
Lower quarter-point of side edge	$+.192\,wa^2$	$+.315\,wa^2$	$+.390\,wa^2$	$+.398\,wa^2$
Total at bottom edge	$.048\,wa^2b$	$.096\,wa^2b$	$.204\,wa^2b$	$.286\,wa^2b$
Total at one fixed side edge	$.226\,wa^2b$	$.202\,wa^2b$	$.148\,wa^2b$	$.107\,wa^2b$
Total at all four edges	$.500\,wa^2b$	$.500\,wa^2b$	$.500\,wa^2b$	$.500\,wa^2b$

*Data are derived by modifying values computed for walls hinged top and bottom.
†This value could not be estimated accurately beyond two decimal places.
‡Negative sign indicates reaction acts in direction of load.

Table VII. Shear at Edges of Slabs Hinged at Top and Bottom



b/a	½	1	2	5	10	Infinity
Mid-point of bottom edge	$+.1407\,wa^2$	$+.2419\,wa^2$	$+.3290\,wa^2$			$+.3333\,wa^2$
Corner at bottom edge	$-.2575\,wa^2$†	$-.4397\,wa^2$	$-.5833\,wa^2$			$-.6000\,wa^2$
Mid-point of fixed side edge	$+.1280\,wa^2$	$+.2582\,wa^2$	$+.3604\,wa^2$			$+.3912\,wa^2$
Lower third-point of side edge	$+.1736\,wa^2$	$+.3113\,wa^2$	$+.4023\,wa^2$			$+.4116\,wa^2$
Lower quarter-point of side edge	$+.1919\,wa^2$	$+.3153\,wa^2$	$+.3904\,wa^2$			$+.3980\,wa^2$
Total at top edge	$.0000\,wa^2b$	$.0052\,wa^2b$	$.0538\,wa^2b$	$.1203\,wa^2b$	$.1435\,wa^2b$	$.1667\,wa^2b$
Total of bottom edge	$.0480\,wa^2b$	$.0960\,wa^2b$	$.1818\,wa^2b$	$.2715\,wa^2b$	$.3023\,wa^2b$	$.3333\,wa^2b$
Total at one fixed side edge	$.2260\,wa^2b$	$.1994\,wa^2b$	$.1322\,wa^2b$	$.0541\,wa^2b$	$.0271\,wa^2b$	$.275\,wa^3$*
Total at all four edges	$.5000\,wa^2b$	$.5000\,wa^2b$	$.5000\,wa^2b$	$.5000\,wa^2b$	$.5000\,wa^2b$	$.5000\,wa^2b$

*Estimated.
†Negative sign indicates reaction acts in direction of load.

Calculation Sheet	Job Number	WBS Number	Page Number	Sheet of

Rev	Date	By	Ck	Title
				Selected Topics — Final Exam / 2012-2013 typical Answer — June, 12, 2013

Q1

a-

$w_d = 0.15 \times 25 + 1.5 = 5.25 \ kN/m^2$

$M_d = \frac{w_d l^2}{10} = \frac{5.25(1.1)^2}{10} = 0.635 \ kN.m/m$

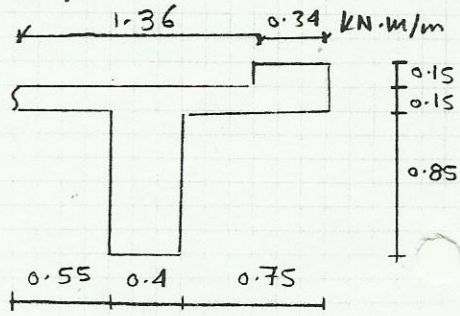
$M_l = 0.8 \frac{3.28 l + 2}{32} P = 0.8 \times \frac{3.28(1.1)+2}{32} \times 72 = 10.094 \ kN.m/m$

$I = \frac{15.24}{l + 38.1} \leq 0.3 \Rightarrow I = \frac{15.24}{1.1 + 38.1} = 0.388 > 0.3 \Rightarrow$ use $I = 0.3$

$M_I = I \times M_L = 0.3(10.094) = 3.0282 \ kN.m/m$

$M_{total} = M_d + M_L + M_I = 0.635 + 10.094 + 3.0282 = 13.757$

b-



$w_d = (1.36 \times 0.15 + 0.34 \times 0.3 + 0.85 \times 0.4) \times 25 + 1.5 \times 1.36 + 1.2 \times 0.34$

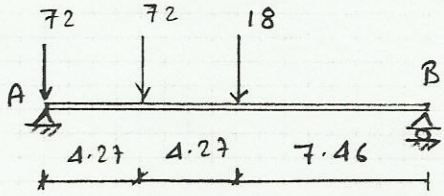
$= 18.598$

$l_{eff.} = \min.\{\text{span c/c, clear span} + h\}$

$= \min.\{16, 15 + 1\} = 16 \ m$

$V_d = \frac{w_d l}{2} = \frac{18.598 \times 16}{2} = 148.784 \ kN$

D.F = 1



$V_l = R_A = 72 + \frac{72 \times 11.73 + 18 \times 7.46}{16}$

$= 133.1775 \ kN$

$I = \frac{15.24}{l + 38.1} \leq 0.3$

$= \frac{15.24}{16 + 38.1} = 0.2817 < 0.3$ o.k

$V_I = I \times V_L = 0.2817 \times 133.1775 = 37.5161 \ kN$

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$V_{total} = V_d + V_l + V_I = 148.784 + 133.1775 + 37.5161 = 319.4776$

$\simeq 319.5$ KN

for interior girder

$w_d = (1.5 \times 0.15 + 0.85 \times 0.4) \times 25 + 1.5 \times 1.5 = 16.375$ KN/m

$V_d = \frac{w_d l}{2} = \frac{16.375\ (16)}{2} = 131$ KN

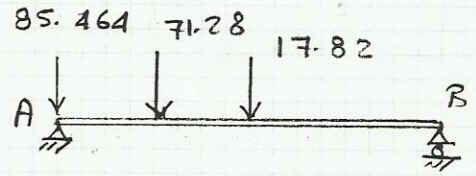
D.F

for rear wheel $= 1.187$

other wheels $DF = 0.66\,S = 0.66 \times 1.5 = 0.99$

$R_A = V_l = 85.464 + \frac{71.28\,(11.73) + 17.82\,(7.46)}{16}$

$= 146.03$ KN



$I = 0.2817$

$V_I = 0.2817\ (146.03) = 41.137$ KN

$V_{total} = V_d + V_l + V_I = 131 + 146.03 + 41.137 = 318.167$ KN

use $V = 319.5$

$d = 850$ mm

$V_c = 0.079\sqrt{f'_c}\ b_w \cdot d = 0.079 \times \sqrt{25} \times 400 \times 850 \times 10^{-3} = 134.3$ KN

use $\phi 16$ bars $a_b = 201$ mm^2

$S = \min\left\{ \frac{d}{2}, 600, \frac{A_v f_s d}{V - V_c}, \frac{3 A_v f_y}{b_w} \right\}$

$= \min\left\{ \frac{850}{2}, 600, \frac{2 \times 201 \times 170 \times 850}{(319.5 - 134.3) \times 10^3}, \frac{3 \times 2 \times 201 \times 400}{400} \right\}$

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Q2 a) $l_{eff.} = \min.\{\text{span c/c}, \text{clear span} + h\} = \min.\{7.5, 7+0.4\}$

$= \min.\{7.5, 7.4\} = 7.4$ m

$w_d = 0.4 \times 25 + 1 = 11$ kN/m²

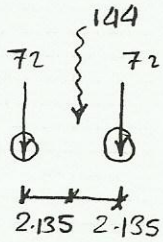
$M_d = \frac{w_d l^2}{8} = \frac{11(7.4)^2}{8} = 75.295$ kN·m/m

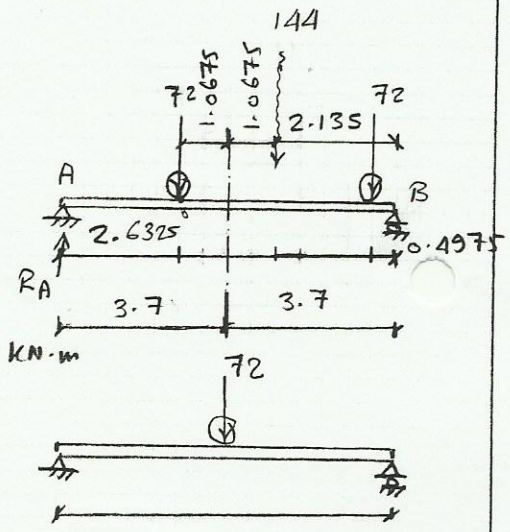
to find M_L

$E = 1.22 + 0.06S \leq 2.14$

$= 1.22 + 0.06 \times 7.4 = 1.664$ m < 2.14 m

Mabs.





$\Sigma M_B = 0$

$R_A \times 7.4 = 144 \times (3.7 - 1.0675)$

$R_A = 51.227$ kN

$M_{abs.} = 51.227 \times 2.6325 = 134.855$ kN·m

or

$M_{abs.} = \frac{72(7.4)}{4} = 133.2$ kN·m

$\therefore M_{LL_{max.}} = \frac{134.855}{1.664} = 81.$ kN·m/m

$I = \frac{15.24}{l+38.1} \leq 0.3 \Rightarrow I = \frac{15.24}{7.4+38.1} = 0.335 > 0.3$

use $I = 0.3$

$M_I = 0.3 \times 81 = 24.3$

$M_{Total} = M_d + M_L + M_I = 75.295 + 81 + 24.3 = 180.595$ kN·m/m

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b) $w_d = 0.6 \times 0.5 \times 25 + 1 \times 0.5 = 8$ KN/m

$$M_d = \frac{w_d l^2}{8} = \frac{8(7.4)^2}{8} = 54.76 \text{ KN.m}$$

$$M_L = 0.1 P l = 0.1 \times 72 \times 7.4 = 53.28 \text{ KN.m}$$

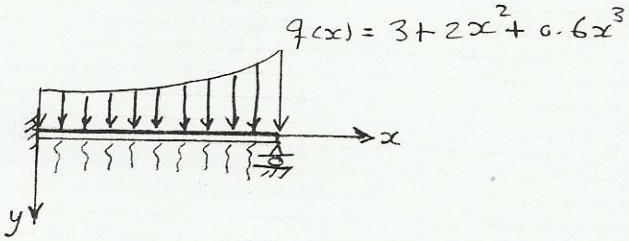
$$M_I = 0$$

$$M_{total} = M_d + M_L = 54.76 + 53.28 = 108.04 \text{ KN.m}$$

Q3

Case I

a)



b) $y_c = e^{-\beta x}(C_1 \cos\beta x + C_2 \sin\beta x) + e^{\beta x}(C_3 \cos\beta x + C_4 \sin\beta x)$

c) $y_p = \frac{3}{k} + \frac{2}{k}x^2 + \frac{0.6}{k}x^3$

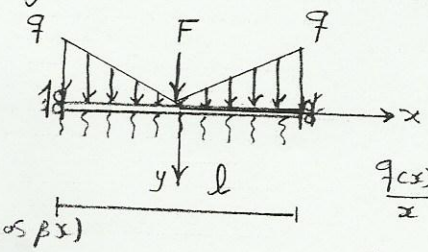
d) $y = y_c + y_p = e^{-\beta x}(C_1 \cos\beta x + C_2 \sin\beta x) + e^{\beta x}(C_3 \cos\beta x + C_4 \sin\beta x) + \frac{3}{k} + \frac{2}{k}x^2 + \frac{0.6}{k}x^3$

BCs @ $x = 0$ $\quad y = 0 \quad y' = 0$

@ $x = l$ $\quad y = 0 \quad M = -EIy'' = 0$

Case II

a)



$$\frac{q(x)}{x} = \frac{q}{l/2}$$

$$q(x) = \frac{2qx}{l}$$

b) $y_c = A_1 \cosh\beta x \cos\beta x + A_2(\cosh\beta x \sin\beta x - \sinh\beta x \cos\beta x) + A_4 \sinh\beta x \sin\beta x$

c) $y_p = \frac{2qx}{kl}$

d) $y = y_c + y_p = A_1 \cosh\beta x \cos\beta x + A_2(\cosh\beta x \sin\beta x - \sinh\beta x \cos\beta x) + A_4 \sinh\beta x \sin\beta x + \frac{2qx}{kl}$

BCs @ $x = l/2$ $\quad V = -EIy''' = 0$

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Q4: $\frac{H^2}{D \cdot t} = \frac{(8)^2}{32 \times 0.5} = 4$

base: hinge
top: free

a- $T = \text{Coeff.}\ \gamma H R = 0.579 \times 10 \times 8 \times 16 = 741.12$ kN/m

$A_s = \frac{T}{f_s} = \frac{741.12 \times 10^3}{100} = 7411.2$ mm²/m $> A_{s\,min.} = 0.0025 A_g = 1250$ mm²/m

use Ø 25 bars ($A_b = 490$ mm²)

Spacing $= 1000 \times \frac{2 \times 490}{7411.2} = 132$ mm

use Ø 25 @ 130 mm c/c E.F

$f_{ct} = \frac{C\ E_s\ A_s + T}{A_g + n\ A_s}$ $\qquad n = \frac{E_s}{E_c} = 8.457$

$= \frac{300 \times 10^{-6} \times 200\,000 \times 7411.2 + 741.12 \times 10^3}{1000 \times 500 + 8.457 \times 7411.2} = 2.1$ MPa < 2.5 MPa

o.k

b-

$n = 8.457$

$r = \frac{f_s}{f_c}$ $\qquad f_c = 0.45 f_c' = 0.45(25) = 11.25$ MPa

$= \frac{150}{11.25} = 13.333$

$k = \frac{n}{n+r} = \frac{8.457}{8.457 + 13.333} = 0.388$

$j = 1 - \frac{k}{3} = 1 - \frac{0.388}{3} = 0.871$

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$M = \text{Coeff.}\ \gamma H^3 = 0.0118 \times 10 \times (8)^3 = 60.416$ kN.m/m

$d_{req'd} = \sqrt{\dfrac{2M}{f_c\ kjb}} = \sqrt{\dfrac{2 \times 60.416 \times 10^6}{11.25 \times 0.388 \times 0.871 \times 442}} = 178.275$

$d_{avail} = 442 > d_{req'd} = 178.275$ mm

o.k

$A_s = \dfrac{M}{f_s\ jd} = \dfrac{60.416 \times 10^6}{150 \times 0.871 \times 442} = 1046.2\ mm^2/m > A_{s\,min.} = 0.0015 A_g$
$= 750\ mm^2/m$

use $\phi 16$ bars ($A_b = 201\ mm^2$)

$\text{Spacing} = 1000 \times \dfrac{201}{1046.2} = 192$

use $\phi 16$ @ 190 mm c/c outer face

$V_{base} = \text{coeff}\ \gamma H^2 = 0.137 \times 10 \times (8)^2 = 87.68$ kN

$V_c = 0.09 \sqrt{f_c'}\ b_w . d = 0.09 \times \sqrt{25} \times 1000 \times 442 \times 10^{-3} = 198.9$ kN

$V_{base} < V_c$ o.k

Q5: $\dfrac{b}{a} = \dfrac{9}{6} = 1.5$

$\dfrac{c}{a} = \dfrac{4.5}{6} = 0.75$

base: hinge
top: hinge

$M_{max.} = 0.036 \times 10 \times (6)^3 = 77.76$ kN.m/m

$n = \dfrac{E_s}{E_c} = \dfrac{200\ 000}{4730\sqrt{25}} = 8.457$

Calculation Sheet	Job Number	WBS Number	Page Number	Sheet of

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$$r = \frac{150}{11.25} = 13.333$$

$$k = \frac{n}{n+r} = \frac{8.457}{8.457+13.333} = 0.388$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.388}{3} = 0.871$$

$$d_{req'd} = \sqrt{\frac{2M}{f_c k j b}} = \sqrt{\frac{2 \times 77.76 \times 10^6}{11.25 \times 0.388 \times 0.871 \times 1000}} = 202.25 \text{ mm}$$

$$d_{avail} = 500 - 50 - \frac{16}{2} = 442 \text{ mm}$$

$d_{avail} > d_{req'd}$ o.k

a- $M_x = 0.032 \times 10 \times (6)^3 = 69.12$ kN.m/m

$$d = 500 - 50 - 16 - \frac{16}{2} = 426 \text{ mm}$$

$$A_s = \frac{M}{f_s j d} = \frac{69.12 \times 10^6}{150 \times 0.871 \times 426} = 1242 \text{ mm}^2/\text{m} > A_{s\,min.} = 0.0015 A_g = 750 \text{ mm}^2/\text{m}$$

use $\phi 16$ bars ($A_b = 201$ mm^2)

$$\text{spacing} = 1000 \times \frac{201}{1242} = 162 \text{ mm}$$

use $\phi 16$ @ 160 mm c/c, vertical reinf., outer face of long wall

b- $M_y = 0.036 \times 10 \times (6)^3 = 77.76$ kN.m/m

$V_2 = P_2$ use $\frac{b}{a} = 1.5$ for shear

$$P_2 = \frac{0.3153 + 0.3904}{2} \times 10 \times (6)^2 = 127.026$$

$$d = 500 - 50 - \frac{16}{2} = 442 \text{ mm}$$

$$d'' = d - \frac{h}{2} = 442 - \frac{500}{2} = 192 \text{ mm}$$

Calculation Sheet	Job Number	WBS Number	Page Number	Sheet of

Rev	Date	By	Ck	Title

$M_s = M - Pd'' = 77.76 - 127.026\,(0.192) = 53.371$ kN.m/m

$$A_s = \frac{M_s}{f_s j d} + \frac{P}{f_s} = \frac{53.371 \times 10^6}{150 \times 0.871 \times 442} + \frac{127.026 \times 10^3}{150} = 1771 \text{ mm}^2/\text{m}$$

$> A_{s\,min.} = 0.0025 A_g = 1250$ mm²/m

use Ø 25 bars ($A_b = 490$ mm²)

$$\text{Spacing} = 1000 \times \frac{490}{1771} = 113.5 \text{ mm}$$

use Ø 25 @ 110 mm c/c

check shear

$V_{max.}$ = for long wall $\frac{b}{a} = 1.5$

$$V_{max.} = \frac{0.3113 + 0.4023}{2} \times 10 \times (6)^2 = 128.448 \text{ kN/m}$$

$$V_c = 0.09\sqrt{f_c'}\, b_w . d = 0.09\sqrt{25} \times 1000 \times 417 \times 10^{-3} = 187.65$$

$V_{max.} < V_c$ o.k

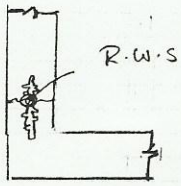
Q6

a) distribution reinf. shall be placed in the bottom of all slabs transverse to the main steel reinforcement its used to provide for the lateral distribution of the concentrated live loads

b) in case $\beta l > \frac{\pi}{4}$

$\beta = \sqrt[4]{\frac{K}{4EI}}$ the beam should be treated as beam on elastic foundation

c) water stop is used in details of all joints in R.C tank



to prevent the leakage of water